# SURFACE FRICTION IN A TURBULENT BOUNDARY LAYER WITH A POSITIVE PRESSURE GRADIENT

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A dependence of the relative friction coefficient on the form parameter of the aerodynamic curvature is proposed on the basis of measurements of the tangential stress on the wall during the flow of a liquid in a diffusor using the electrodiffusion method. The intensity of fluctuations of the tangential stress at the wall and the instant of appearance of reverse flows are investigated. The results of measurement of the friction coefficient by the electrodiffusion method and Clauser method are compared.

For the turbulent boundary layer on a two-dimensional plate a large amount of reliable experimental data on surface friction is available, obtained by the floating-element method [1]. The reliability of such measurements is inadequate for flow with a pressure gradient.

Other methods are also not reliable, and the data corresponding to these are scanty. In [2] the surface friction in a flow with a pressure gradient has been measured by the heat-element method. There are a number of studies, in which the tangential stress at the wall has been determined with the use of the so-called universal surface distribution of the velocity, whose numerical coefficients are taken from the results of measurements on a two-dimensional plate and are correlated only on the basis of experiments [2]. An appropriate review is given in [3].

Below, data are presented that were obtained from direct measurements of friction at the walls of a diffusor by the electrodiffusion method [4, 5].

The operating segment was made in the form of a plane diffusor with the input cross section 40 × 100 $mm^2$, the expansion angle 8°, and length 440 mm. Before the operating segment there was a forechamber with tubular convergent channel with waisting coefficient equal to three and a preconnected segment representing a plane channel of cross section 40 × 100 $mm^2$ and length 440 mm serving for the development of the initial thickness of the boundary layer at the entrance to the diffusor. The schematic diagram of the operating segment is shown in Fig. 1. A total of seven electrodiffusion sensors were placed along the central line of one of the diverging walls of the diffusor. The distance between adjacent sensors was 55 mm, and the first sensor was placed at a distance of 440 mm from the entrance. The sensitive element of the sensor was the end face of a platinum wire of 0.5 mm diameter or a plate of 0.02 × 0.2 $mm^2$ cross section. The operating liquid was a 0.005 N solution of potassium ferric- and ferrocyanide and 0.5 N sodium hydroxide in distilled water. For ensuring the constancy of the physical properties the temperature of the liquid was maintained constant in the range 25 ± 0.2°C.

The magnitude of the tangential stress at the wall $\tau_0$ was estimated from the formula [5]

$$\tau_0 = \frac{1.87\mu}{F^3 l^2 h^3 D^2 c_0^3} I^3 = A I^3 \tag{1}$$

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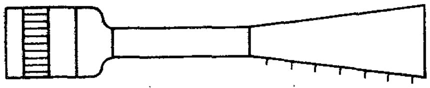

Fig. 1

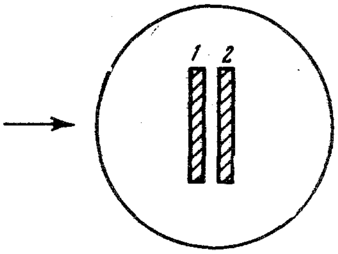

Fig. 2

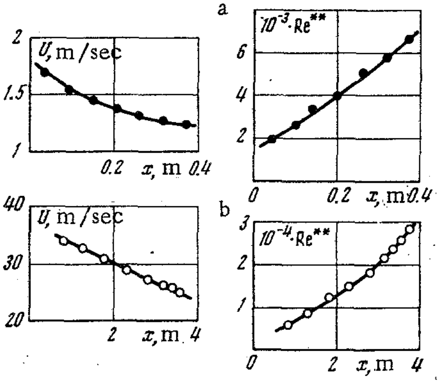

Fig. 3

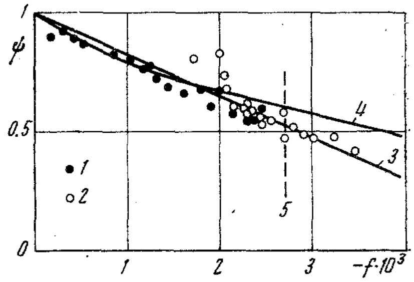

Fig. 4

where μ is the dynamic viscosity, F is Faraday number, $l$, h are the longitudinal and transverse dimensions of the sensor, D is the diffusion coefficient, $c_0$ is the concentration of ferricyanide ions, and I is the sensor current.

The computational value of coefficient A was determined from measurements for a fully developed turbulent flow in a plane channel. The standard value of the tangential stress was calculated from the Blausius formula. The difference of the calibration from the exact estimate by Formula (1) did not exceed 7%.

Since the presence of strong fluctuations of the velocity and the tangential stress at the wall are typical for turbulent flow in a diffusor, the average value of the tangential stress was computed from the formula

$$\bar{\tau}_0 = A\bar{I}^3 \cong (\bar{I}^3 + 3\bar{I}\overline{I'^2}) \tag{2}$$

The average value of current $\bar{I}$ was determined by averaging the current readings from the sensor with 0.5 mm diameter, for which a high stability of the readings in time is characteristic. The mean-square fluctuations of current I' were measured by a sensor having a dimension of 0.02 mm along the flow, which practically eliminated the influence of the signal frequency on the characteristics of the instrument [4]. The instantaneous value of the signal from this sensor was squared and then averaged.

The relative intensity of the turbulent fluctuations of the tangential stress $\tau_0'$ at the wall was estimated from a formula similar to that used in thermoanemometry [6],

$$\sqrt{\overline{\tau_0'^2}}/\bar{\tau}_0 = 3\sqrt{\overline{I'^2}}/\bar{I} \tag{3}$$

The flow velocity was measured by a pitot tube with the input cross section equal to 0.3 × 0.6 mm². The tube was placed perpendicular to the wall. The measurements were made for the velocity of the impinging flow equal to 1.1, 1.53, and 2.24 m/sec. The degree of turbulence of the flow at the exit from the convergent channel was 2.2-2.5%. This quantity was measured with an electrodiffusion velocity sensor [5].

For recording the instant of appearance of reverse flows a sensor of special construction was used; it consisted of two closely placed cathodes (Fig. 2). For the direction of the flow shown in the figure, sensor 2 indicated smaller values of the current than sensor 1 due to the screening action of the diffusion layer of the first sensor. In the case of development of the reverse flow the smaller value of the current was shown by sensor 1. The sensor currents were compared by a special electronic circuit which also permitted the determination of the time of existence of the reverse flow. In order to increase the speed of response for recording short surges of reverse flow at the instant of their generation, the overall dimension of the twin sensor was made as small as possible (50 μm).

The distributions of the velocity at the outer boundary of the boundary layer U and Reynolds number Re** are shown in Fig. 3a; Re** was obtained from the thickness of the momentum loss δ** along the length of the operating segment for the velocity at the entrance cross section equal to 1.53 m/sec. The same quantities obtained from the experiments [2] are shown in Fig. 3b.

The results of measurements of the tangential stress are shown in Fig. 4 in the form of the dependence of the relative friction coefficient

$$\psi = (C_f / C_{f_0})_{\mathrm{Re}^{**}} = \mathrm{const}$$

where $C_{f_0}$ is the coefficient of friction at a plane plate, on the form parameter of aerodynamic curvature

$$f = (\delta^{**}/U)(dU/dx).$$

The results of our experiments, as well as those of the experiments in [2], are well approximated by the linear dependence

$$\psi = 1 + 1.77 \cdot 10^2 f \qquad (4)$$

The symbols used in Fig. 4 are: 1) data from [2], 2) our data, 3) dependence according to equation(4), 4) computation according to [7]; the dashed line 5 corresponds to the instant of appearance of reverse flows.

The results of the computation according to [7] describe quite well the experimental data up to values of the form parameter $f$ equal to $2.6 \cdot 10^{-3}$.

Using the twin sensor described above the appearance of small duration surges of reverse flow was recorded for $f \approx 2.7 \cdot 10^{-3}$. The time of existence of the reverse flow did not exceed 2%; thus, the presence of surges of reverse flow could not significantly affect the correctness of the measurements of the tangential stress at the wall.

The relation between the form parameters H and $f$ is shown in Fig. 5 based on data of [2] and our data; here H is the ratio of the displacement thickness to the thickness of the momentum loss. In the first approximation the relation between these quantities can be described by the linear dependence

$$H = 1.3 - 1.77 \cdot 10^2 f \qquad (5)$$

In Fig. 5 points 1 correspond to data of [2]; 2, to our data; and 3, to the dependence computed from equation (5).

The results of measurement of the relative intensity of the turbulent fluctuations of the tangential stress at the wall $\sqrt{\overline{\tau'^2}}/\overline{\tau}$ are shown in Fig. 6 as a function of the form parameter $f$. With the increase of the absolute value of $f$ the intensity of fluctuations of the tangential stress increases up to 0.7, whereas in nongradient flow it is close to 0.3 as shown in [8]. The presence of fluctuations of large intensity is accompanied by the formation of "tongues" of reverse flow, which appear at $f = -2.7 \cdot 10^{-3}$. In the presence of large fluctuations the second term in (2) must be taken into consideration, which comprised up to 15% of the average value in the present experiments. In Fig. 4 the results of [2] lie, on the average, below dependence (4); this may be accounted for by the neglect of the second term of (2) in the measurements by the heat-element method.

Most of the results on the measurement of surface friction have been obtained by Clauser method based on the universal velocity profile [9]. A comparison of the results of measurement of the friction coefficient by the electrodiffusion method $C_f$ and by Clauser method $C_{f_c}$, which is shown in Fig. 7, indicates that for gradient flows the use of direct methods of measuring friction is desirable.

It has been observed in [2] and also in other experiments (see [3]) that the momentum equation is not satisfied in the case of gradient flows. This phenomenon was observed in our experiments also.

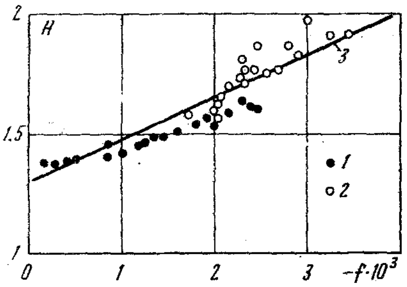

Fig. 5

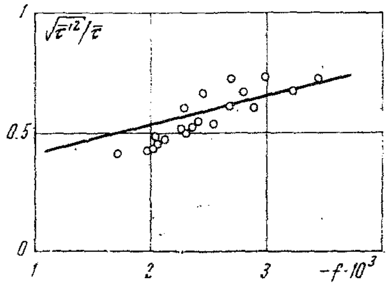

Fig. 6

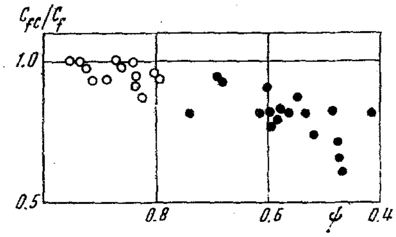

Fig. 7

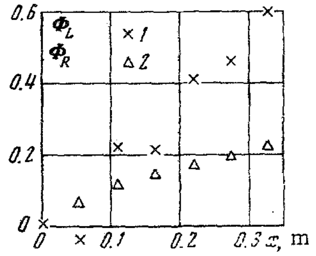

Fig. 8

Figure 8 shows the left and right sides of the momentum equation $\Phi_L$ (point 1) and $\Phi_R$ (point 2) integrated over x in the following way:

$$\Phi_L = \frac{U^2\delta^{**}}{(U^2\delta^{**})_0} - 1 + \frac{1}{2}\int_{x_0}^{x} \frac{\delta^*}{\delta_0^{**}}\, d\left(\frac{U^2}{U_0{}^2}\right)$$

$$\Phi_R = \int_{x_0}^{x} \left(\frac{v_*}{U_0}\right)^2 d\left(\frac{x}{\delta_0^{**}}\right) \qquad (6)$$

where $\delta^*$ is the displacement thickness, $v^* = \sqrt{\tau_0/\rho}$ is the dynamic velocity, $\rho$ is the density of the liquid, and the index 0 denotes quantities at the point corresponding to the first sensor $x = x_0$, for velocity of the oncoming flow equal to 1.53 m/sec.

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